Solutions to Quiz 1, ECED 3300

Problem 1

1)

$$\nabla F = \mathbf{a}_r \partial_r F + \tfrac{1}{r}\mathbf{a}_\theta \partial_\theta F = -\tfrac{2\cos\theta}{r^3}\mathbf{a}_r - \tfrac{\sin\theta}{r^3}\mathbf{a}_\theta,$$

Thus,

$$\nabla F = -\frac{2\cos\theta \mathbf{a}_r + \sin\theta \mathbf{a}_\theta}{r^3}.$$

2)

$$(\nabla F)_\parallel = \mathbf{a}_z(\mathbf{a}_z \cdot \nabla F) = -\frac{1}{r^3}\left[2\cos\theta(\mathbf{a}_r \cdot \mathbf{a}_z) + \sin\theta(\mathbf{a}_\theta \cdot \mathbf{a}_z)\right]\mathbf{a}_z,$$

Using the formula sheet, $\mathbf{a}_r \cdot \mathbf{a}_z = \cos\theta$ and $\mathbf{a}_\theta \cdot \mathbf{a}_z = -\sin\theta$, yields the final answer

$$(\nabla F)_\parallel = -\frac{\mathbf{a}_z}{r^3}(2\cos^2\theta - \sin^2\theta).$$

Problem 2

Apply Gauss's theorem,

$$\oint_S d\mathbf{S} \cdot \mathbf{A} = \int_v dv \nabla \cdot \mathbf{A}.$$

The divergence does not depend on the coordinate system. Hence, using the Cartesian coordinates,

$$\nabla \cdot \mathbf{A} = \partial_x A_x + \partial_y A_y + \partial_z A_z = \partial_x z^2 + \partial_y x^2 + \partial_z y^2 = 0.$$

Finally,

$$\oint_S d\mathbf{S} \cdot \mathbf{A} = \int_v dv \nabla \cdot \mathbf{A} = 0.$$

Problem 3

As the path is **not** closed, Stokes's theorem **cannot** be applied. Use the definition instead.

$$d\mathbf{l} = \mathbf{a}_\phi \rho d\phi = \mathbf{a}_\phi d\phi.$$

Thus,

$$\int_L d\mathbf{l} \cdot \mathbf{B} = -\int_0^\pi d\phi(\mathbf{a}_\phi \cdot \mathbf{a}_x).$$

Using the formula sheet,

$$-\mathbf{a}_x = \mathbf{a}_\phi \sin\phi - \mathbf{a}_\rho \cos\phi,$$

implying that

$$\int_L d\mathbf{l} \cdot \mathbf{B} = -\int_0^\pi d\phi(\mathbf{a}_\phi \cdot \mathbf{a}_x) = \int_0^\pi d\phi(\mathbf{a}_\phi \cdot \mathbf{a}_\phi)\sin\phi = 2.$$